

Influence of rare regions on quantum phase transition in antiferromagnets with hidden degrees of freedom

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Abstract

The effects of rare regions on the critical properties of quantum antiferromagnets with hidden degrees of freedom within the renormalization group is discussed. It is shown that for “constrained” systems the stability range on the phase diagram remains the same as in the mean-field theory while for “unconstrained” systems the stability range is effectively decreased.

Key words: Magnetically ordered materials. Point defects. Phase transitions.

1 Introduction

The influence of quenched disorder on the critical properties of itinerant quantum magnets is a very interesting problem in phase transition theory. Particular interest was given to locally ordered spatial regions (“rare regions”) which are formed in the presence of quenched disorder even when the bulk system is still in the paramagnetic phase [1]. The conventional theory [2] ignores these

¹ Partially supported by Russian Foundation for Basic Research (97-02-17315), Russian Federation.

rare regions. For classical magnets, it has recently been shown that the rare regions induce a new term in the action which breaks the replica symmetry and that the conventional theory is unstable with respect to this perturbation [3].

The problem of rare regions for the case of quantum phase transitions has been considered in Refs. [4,5]. These authors showed that the rare regions destroy the fixed point found in the conventional theory of itinerant antiferromagnets [6] and, in contrast, critical behavior of itinerant ferromagnets [7] is unaffected by the rare regions due to an effective long-range interaction between the order parameter fluctuations.

The conventional theory of a random- T_c quantum antiferromagnet with the hidden degrees of freedom on which various constraints are imposed has been considered in Ref. [8] and it has been shown that for “constrained” systems the stability range on the phase diagram remains the same as in the mean-field theory while for “unconstrained” systems the stability range is effectively decreased. In this paper we study the effects of rare regions on the quantum phase transition in antiferromagnets with hidden degrees of freedom.

2 Renormalization group equations

Let us consider a disordered itinerant quantum antiferromagnet in which the n -component vector order parameter $\mathbf{S}(\mathbf{r}, \tau)$ is coupled with the scalar non-fluctuating parameter $y(\mathbf{r}, \tau)$. We use an action

$$\begin{aligned}
S = & \frac{1}{2} \int d\mathbf{r}_1 d\mathbf{r}_2 \int_0^{1/T} d\tau d\tau' [t(\mathbf{r}_1 - \mathbf{r}_2, \tau - \tau') \\
& + \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\tau - \tau') \delta t(\mathbf{r})] \mathbf{S}(\mathbf{r}_1, \tau) \mathbf{S}(\mathbf{r}_2, \tau') \\
& + \lambda_s \int d\mathbf{r} \int_0^{1/T} d\tau [\mathbf{S}(\mathbf{r}, \tau) \mathbf{S}(\mathbf{r}, \tau)]^2 \\
& + \mu \int d\mathbf{r} \int_0^{1/T} d\tau y(\mathbf{r}, \tau) [\mathbf{S}(\mathbf{r}, \tau)]^2 \\
& + \frac{1}{2} \beta \int d\mathbf{r} \int_0^{1/T} d\tau [y(\mathbf{r}, \tau)]^2.
\end{aligned} \tag{1}$$

Here the function $t(\mathbf{r}, \tau)$ is the Fourier-transform of the two-point interaction of a quantum antiferromagnet [9,6]

$$t(\mathbf{q}, \omega_n) = t + \mathbf{q}^2 + \omega_n, \quad (2)$$

where t denotes the distance from the quantum critical point, \mathbf{q} is the wave vector and ω_n is the Matsubara frequency.

As the given model is phenomenological, we suppose that the coupling constant of the order parameter with the nonfluctuating parameter is purely imaginary as it occurs in the Hubbard model [10–12]. It is easy to consider the case with a real coupling constant. The distinction between these two cases consists only in the definition of the part of an appropriate phase space which corresponds to a real physical model. In contrast to the classical phase transition the order parameter depends on both the d -dimensional vector of space \mathbf{r} and imaginary time τ [9].

In order to write an action of the conventional theory the replica trick should be used and then an integration over random variables should be performed. For simplicity we consider the case when the coefficient of the quadratic term in the order parameter in the action, $\delta t(\mathbf{r})$, is the fluctuating Gaussian variable with zero mean and variance Δ . Finally we obtain an action which has homogeneous saddle points only [8].

To incorporate rare regions into the theory we need a different approach. In analogy to Refs. [3–5], we consider inhomogeneous saddle-point solutions for a fixed realization of disorder. The partition function can be written as the sum of all contributions obtained from the vicinity of each saddle-point [5]. Then we can average over disorder by means of the replica trick. We finally obtain the following effective action

$$\begin{aligned} S_{\text{eff}} = & \frac{1}{2} \int d\mathbf{r}_1 d\mathbf{r}_2 \int_0^{1/T} d\tau d\tau' t(\mathbf{r}_1 - \mathbf{r}_2, \tau - \tau') \sum_{\alpha} \mathbf{S}^{\alpha}(\mathbf{r}_1, \tau) \mathbf{S}^{\alpha}(\mathbf{r}_2, \tau') \\ & + \lambda_s \int d\mathbf{r} \int_0^{1/T} d\tau \sum_{\alpha} [\mathbf{S}^{\alpha}(\mathbf{r}, \tau) \mathbf{S}^{\alpha}(\mathbf{r}, \tau)]^2 \\ & - \sum_{\alpha, \beta} (\Delta + x \delta_{\alpha, \beta}) \int d\mathbf{r} \int_0^{1/T} d\tau d\tau' [\mathbf{S}^{\alpha}(\mathbf{r}, \tau) \mathbf{S}^{\alpha}(\mathbf{r}, \tau)] \times \\ & [\mathbf{S}^{\beta}(\mathbf{r}, \tau') \mathbf{S}^{\beta}(\mathbf{r}, \tau')] \\ & + \mu \int d\mathbf{r} \int_0^{1/T} d\tau \sum_{\alpha} y^{\alpha}(\mathbf{r}, \tau) [\mathbf{S}^{\alpha}(\mathbf{r}, \tau)]^2 \end{aligned}$$

$$+\frac{1}{2}\beta \int d\mathbf{r} \int_0^{1/T} d\tau \sum_{\alpha} [y^{\alpha}(\mathbf{r}, \tau)]^2. \quad (3)$$

Here α and β are replica indices. The x -term is generated by taking into account the inhomogeneous saddle points. The conventional theory misses this term.

Let us assume the temperature T to be equal zero and use double ϵ expansion [6] according to which the space dimensionality is equal $4 - \epsilon$ and the dimensionality of imaginary time is ϵ_{τ} . Of course, for the real physical case we have $\epsilon = \epsilon_{\tau} = 1$.

Defining $\bar{x} = xT^{-\epsilon_{\tau}}$, and putting $T = 0$, we obtain the following renormalization group flow equations in the one-loop approximation

$$\frac{du}{dl} = (\epsilon - 2\epsilon_{\tau})u - 4(n+8)u^2 + 6u\Delta, \quad (4)$$

$$\frac{d\Delta}{dl} = \epsilon\Delta - 8(n+2)u\Delta + 4\Delta^2 + 8n\Delta\bar{x}, \quad (5)$$

$$\frac{d\bar{x}}{dl} = (\epsilon - 2\epsilon_{\tau})\bar{x} - 8(n+2)u\bar{x} + 4n\bar{x}^2 + 6\Delta\bar{x}, \quad (6)$$

$$\frac{dz}{dl} = (\epsilon - 2\epsilon_{\tau})z - 8(n+2)uz - 2nz^2 + 2z\Delta, \quad (7)$$

$$\frac{dw}{dl} = (\epsilon - 2\epsilon_{\tau})w - 8(n+2)uw - 2nw^2 - 4nzw + 2w\Delta, \quad (8)$$

where $l = \ln b$ with b the scale parameter, and we have scaled $K_4 u \rightarrow u$, $K_4 \Delta \rightarrow \Delta$, $K_4 \bar{x} \rightarrow \bar{x}$, $K_4 z \rightarrow z$, $K_4 w \rightarrow w$ with $K_4 = 1/8\pi^2$. We also denote here

$$u = \lambda_s - \frac{\mu^2}{2\beta}, \quad (9)$$

$$z = \frac{\mu^2}{\beta} - \frac{\mu_0^2}{\beta_0}, \quad (10)$$

$$w = \frac{\mu_0^2}{\beta_0}. \quad (11)$$

In equations (4)-(8) we separate the coefficient of the nonfluctuating parameter $y(\mathbf{q} = 0)$ from $y(\mathbf{q} \neq 0)$ because of its possible role in constraining systems [10,13].

3 Fixed points

Before discussing the renormalization group analysis we consider the mean-field theory result. After integrating over the nonfluctuating parameter in the equation for the partition function we can obtain a new effective action in terms of the order parameter $\mathbf{S}(\mathbf{r})$. In the mean-field approximation for this effective action the boundaries of a stability range can be easily found. It is convenient to introduce new notations for the coupling constants of the effective action as

$$\lambda_c^{(0)} = \mu_0^2/2\beta_0, \quad \lambda_c^{(1)} = \mu^2/2\beta. \quad (12)$$

For the unconstrained system $\lambda_c^{(0)} = \lambda_c^{(1)} \equiv \lambda_c$ (or $z = 0$) while the boundaries of the stability range correspond to the equations of lines

$$\lambda_s - \lambda_c^{(0)} = 0, \quad \lambda_c^{(0)} = 0. \quad (13)$$

For the constrained system $\lambda_c^{(0)} = 0$ (or $w = 0$) while the boundaries of the stability range can be written as

$$\lambda_s = 0, \quad \lambda_c^{(1)} = 0. \quad (14)$$

The stability ranges are represented on planes $(\lambda_c - \lambda_s)$ (Fig. 1) and $(\lambda_c^{(1)} - \lambda_s)$ (Fig. 2).

The characteristic feature of flow equations is the closed system of three equations (4)-(6). Within the notations this set of equations coincides with the appropriate set of equations of Ref. [4] where however the nonfluctuating degrees of freedom were not taken into account. It is easy to find all eight fixed points (4)-(6). Four of the fixed points have a zero fixed-point value of \bar{x}^* , $\bar{x}^* = 0$. The other four fixed points have $\bar{x}^* \neq 0$.

Using these fixed points we can find other fixed points from the flow equations (7), (8). There are the sixteen fixed points with $\bar{x}^* = 0$ (Table 1) studied before in Ref. [8]. The other sixteen fixed points have $\bar{x}^* \neq 0$ (Table 2). As can be seen from Table 1 we can separate all fixed points on groups consisting of two fixed points, for example group of Gaussian points (G) and group of renormalized Gaussian points (RG). Each group has its proper set of critical exponents.

Such classification of fixed points was used for description of phase transition in classical systems [10,14,15] where for renormalized fixed points the critical exponents can be found according to Fisher [16] via critical exponents of

Table 1
Fixed points with $\bar{x}^* = 0$

	u^*	Δ^*	\bar{x}^*	z^*	w^*
G_1	0	0	0	0	$\frac{\epsilon-2\epsilon_\tau}{2n}$
G_2	0	0	0	0	0
RG_1	0	0	0	$\frac{\epsilon-2\epsilon_\tau}{2n}$	0
RG_2	0	0	0	$\frac{\epsilon-2\epsilon_\tau}{2n}$	$-\frac{\epsilon-2\epsilon_\tau}{2n}$
H_1	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	0	0	$\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$
H_2	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	0	0	0
RH_1	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	0	$\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$	0
RH_2	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	0	$\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$	$-\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$
U_1	0	$-\frac{\epsilon}{4}$	0	0	$\frac{\epsilon-4\epsilon_\tau}{4n}$
U_2	0	$-\frac{\epsilon}{4}$	0	0	0
RU_1	0	$-\frac{\epsilon}{4}$	0	$\frac{\epsilon-4\epsilon_\tau}{4n}$	0
RU_2	0	$-\frac{\epsilon}{4}$	0	$\frac{\epsilon-4\epsilon_\tau}{4n}$	$-\frac{\epsilon-4\epsilon_\tau}{4n}$
R_1	$\frac{\epsilon+4\epsilon_\tau}{16(n-1)}$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{8(n-1)}$	0	0	$\frac{(n-4)\epsilon-12n\epsilon_\tau}{8n(n-1)}$
R_2	$\frac{\epsilon+4\epsilon_\tau}{16(n-1)}$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{8(n-1)}$	0	0	0
RR_1	$\frac{\epsilon+4\epsilon_\tau}{16(n-1)}$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{8(n-1)}$	0	$\frac{(n-4)\epsilon-12n\epsilon_\tau}{8n(n-1)}$	0
RR_2	$\frac{\epsilon+4\epsilon_\tau}{16(n-1)}$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{8(n-1)}$	0	$\frac{(n-4)\epsilon-12n\epsilon_\tau}{8n(n-1)}$	$-\frac{(n-4)\epsilon-12n\epsilon_\tau}{8n(n-1)}$

appropriate non-renormalized values

$$\alpha_{\text{renorm}} = -\frac{\alpha}{1-\alpha}, \quad \nu_{\text{renorm}} = \frac{\nu}{1-\alpha}. \quad (15)$$

The eigenvalues λ_i ($i = 1, \dots, 5$) of flow equations linearized about the fixed point (4)-(8) are indicated in Table 3 and Table 4.

It should be noted that eigenvalues λ_1 and λ_2 for random (R) and renormalized random (RR) fixed points for $n > 1$ are complex.

Due to the relation

$$u = \lambda_s - \lambda_c^{(1)}, \quad (16)$$

the fixed points with $\Delta^* = 0$ and $\bar{x}^* = 0$ align on parallel lines:

- (1) $\lambda_s = \lambda_c$ for the unconstrained system ($z = 0$);
- (2) $\lambda_s = \lambda_c^{(1)}$ for the constrained system ($w = 0$);

Table 2

Fixed points with $\bar{x}^* \neq 0$

	u^*	Δ^*	\bar{x}^*	z^*	w^*
1	0	0	$-\frac{\epsilon-2\epsilon_\tau}{4n}$	0	$\frac{\epsilon-2\epsilon_\tau}{2n}$
2	0	0	$-\frac{\epsilon-2\epsilon_\tau}{4n}$	0	0
3	0	0	$-\frac{\epsilon-2\epsilon_\tau}{4n}$	$\frac{\epsilon-2\epsilon_\tau}{2n}$	0
4	0	0	$-\frac{\epsilon-2\epsilon_\tau}{4n}$	$\frac{\epsilon-2\epsilon_\tau}{2n}$	$-\frac{\epsilon-2\epsilon_\tau}{2n}$
5	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	$\frac{(n-4)(\epsilon-2\epsilon_\tau)}{4n(n+8)}$	0	$\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$
6	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	$\frac{(n-4)(\epsilon-2\epsilon_\tau)}{4n(n+8)}$	0	0
7	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	$\frac{(n-4)(\epsilon-2\epsilon_\tau)}{4n(n+8)}$	$\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$	0
8	$\frac{\epsilon-2\epsilon_\tau}{4(n+8)}$	0	$\frac{(n-4)(\epsilon-2\epsilon_\tau)}{4n(n+8)}$	$\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$	$-\frac{(\epsilon-2\epsilon_\tau)(4-n)}{2n(n+8)}$
9	0	$\frac{-\epsilon+4\epsilon_\tau}{8}$	$-\frac{\epsilon+4\epsilon_\tau}{16n}$	0	$\frac{3\epsilon-4\epsilon_\tau}{8n}$
10	0	$\frac{-\epsilon+4\epsilon_\tau}{8}$	$-\frac{\epsilon+4\epsilon_\tau}{16n}$	0	0
11	0	$\frac{-\epsilon+4\epsilon_\tau}{8}$	$-\frac{\epsilon+4\epsilon_\tau}{16n}$	$\frac{3\epsilon-4\epsilon_\tau}{8n}$	0
12	0	$\frac{-\epsilon+4\epsilon_\tau}{8}$	$-\frac{\epsilon+4\epsilon_\tau}{16n}$	$\frac{3\epsilon-4\epsilon_\tau}{8n}$	$-\frac{3\epsilon-4\epsilon_\tau}{8n}$
13	$\frac{\epsilon+4\epsilon_\tau}{8(10-n)}$	$\frac{(n-4)\epsilon+24\epsilon_\tau}{4(10-n)}$	$\frac{(n-4)(\epsilon+4\epsilon_\tau)}{8n(10-n)}$	0	$\frac{(4-n)3\epsilon-(n+8)4\epsilon_\tau}{4n(10-n)}$
14	$\frac{\epsilon+4\epsilon_\tau}{8(10-n)}$	$\frac{(n-4)\epsilon+24\epsilon_\tau}{4(10-n)}$	$\frac{(n-4)(\epsilon+4\epsilon_\tau)}{8n(10-n)}$	0	0
15	$\frac{\epsilon+4\epsilon_\tau}{8(10-n)}$	$\frac{(n-4)\epsilon+24\epsilon_\tau}{4(10-n)}$	$\frac{(n-4)(\epsilon+4\epsilon_\tau)}{8n(10-n)}$	$\frac{(4-n)3\epsilon-(n+8)4\epsilon_\tau}{4n(10-n)}$	0
16	$\frac{\epsilon+4\epsilon_\tau}{8(10-n)}$	$\frac{(n-4)\epsilon+24\epsilon_\tau}{4(10-n)}$	$\frac{(n-4)(\epsilon+4\epsilon_\tau)}{8n(10-n)}$	$\frac{(4-n)3\epsilon-(n+8)4\epsilon_\tau}{4n(10-n)}$	$-\frac{(4-n)3\epsilon-(n+8)4\epsilon_\tau}{4n(10-n)}$

(3) $\lambda_s - \lambda_c = (\epsilon - 2\epsilon_\tau)/4K_4(n+8)$ for the unconstrained system ($z = 0$);(4) $\lambda_s - \lambda_c^{(1)} = (\epsilon - 2\epsilon_\tau)/4K_4(n+8)$ for the constrained system ($w = 0$).

It is easy to see that for the quantum phase transition ($\epsilon_\tau \neq 0$) in the case when the inequality $\epsilon < 2\epsilon_\tau$ is satisfied the fixed points lying on the two last lines are not situated in the stability range of the effective action (Fig. 1 and Fig. 2). Thus the stability range for the quantum phase transition in the constrained system remains such as in the mean-field theory in contrast with the classical phase transition where these fixed points are situated in the stability range and cause an effective reduction of this range [11,12]. However for the unconstrained system there is the unstable fixed point G_1 on the boundary of the stability range. Its critical exponents coincide with critical exponents of the point G_2 . Analogous to the classical phase transition a separatrix going out this point divides the stability range in two regions. One is the region of accessibility of the fixed point G_2 and the other is the region of phase space where flow trajectories approach to the “invariant” line $G_2 - G_1$.

Correspondingly, the fixed points with $\Delta^* \neq 0$ and $\bar{x}^* = 0$ are situated along parallel lines:

Table 3

Eigenvalue of fixed points with $\bar{x}^* = 0$ ($\lambda_{1,2}^R = \frac{1}{8(n-1)}\{-3n\epsilon + 4(n-4)\epsilon_\tau \pm [(5n-8)^2\epsilon^2 + 24(16-12n-n^2)\epsilon\epsilon_\tau + 48(16-8n-5n^2)\epsilon_\tau^2]^{1/2}\}$)

	λ_1	λ_2	λ_3	λ_4	λ_5
G_1	$\epsilon - 2\epsilon_\tau$	ϵ	$\epsilon - 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$
G_2	$\epsilon - 2\epsilon_\tau$	ϵ	$\epsilon - 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$
RG_1	$\epsilon - 2\epsilon_\tau$	ϵ	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$
RG_2	$\epsilon - 2\epsilon_\tau$	ϵ	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$
H_1	$-\epsilon + 2\epsilon_\tau$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$
H_2	$-\epsilon + 2\epsilon_\tau$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$
RH_1	$-\epsilon + 2\epsilon_\tau$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$
RH_2	$-\epsilon + 2\epsilon_\tau$	$\frac{(4-n)\epsilon+4(n+2)\epsilon_\tau}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon-2\epsilon_\tau)}{n+8}$
U_1	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$-\epsilon$	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$\frac{1}{2}\epsilon - 2\epsilon_\tau$	$-\frac{1}{2}\epsilon + 2\epsilon_\tau$
U_2	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$-\epsilon$	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$\frac{1}{2}\epsilon - 2\epsilon_\tau$	$\frac{1}{2}\epsilon - 2\epsilon_\tau$
RU_1	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$-\epsilon$	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$-\frac{1}{2}\epsilon + 2\epsilon_\tau$	$-\frac{1}{2}\epsilon + 2\epsilon_\tau$
RU_2	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$-\epsilon$	$-\frac{1}{2}\epsilon - 2\epsilon_\tau$	$-\frac{1}{2}\epsilon + 2\epsilon_\tau$	$\frac{1}{2}\epsilon - 2\epsilon_\tau$
R_1	λ_1^R	λ_2^R	$\frac{(4-n)(\epsilon+4\epsilon_\tau)}{4(n-1)}$	$\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$	$-\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$
R_2	λ_1^R	λ_2^R	$\frac{(4-n)(\epsilon+4\epsilon_\tau)}{4(n-1)}$	$\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$	$\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$
RR_1	λ_1^R	λ_2^R	$\frac{(4-n)(\epsilon+4\epsilon_\tau)}{4(n-1)}$	$-\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$	$-\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$
RR_2	λ_1^R	λ_2^R	$\frac{(4-n)(\epsilon+4\epsilon_\tau)}{4(n-1)}$	$-\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$	$\frac{(n-4)\epsilon-12n\epsilon_\tau}{4(n-1)}$

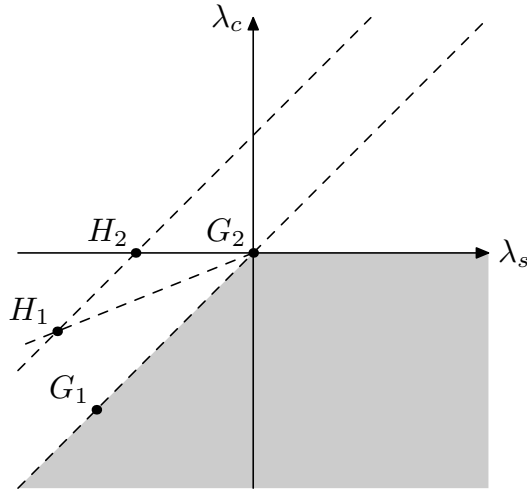
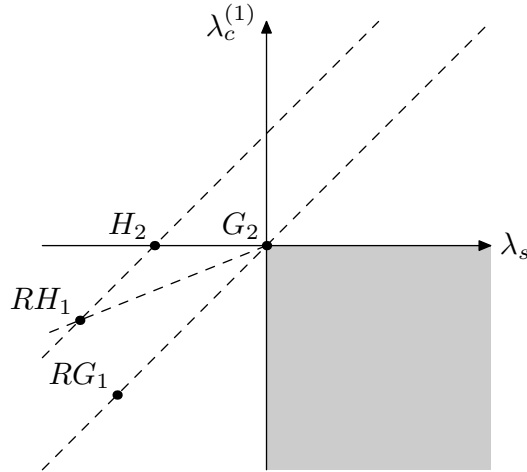
Fig. 1. Phase diagram for the unconstrained system ($\Delta^* = 0$, $\bar{x}^* = 0$)

Table 4

Eigenvalue of fixed points with $\bar{x}^* \neq 0$ ($\lambda_{2,3}^{(9)} = \frac{1}{8}\{4\epsilon_\tau - 3\epsilon \pm [25\epsilon^2 - 24\epsilon\epsilon_\tau - 240\epsilon_\tau^2]^{1/2}\}$ and $\lambda_{1,2}^{(13)} = -\frac{1}{4(10-n)}\{(16-n)\epsilon + 4(n-4)\epsilon_\tau \pm [9(8-n)^2\epsilon^2 + 24(n^2 - 4n - 48)\epsilon\epsilon_\tau + 16(n^2 + 40n - 464)\epsilon_\tau^2]^{1/2}\}$)

	λ_1	λ_2	λ_3	λ_4	λ_5
1	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 4\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$
2	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 4\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$
3	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 4\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$
4	$\epsilon - 2\epsilon_\tau$	$-\epsilon + 4\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$-\epsilon + 2\epsilon_\tau$	$\epsilon - 2\epsilon_\tau$
5	$-\epsilon + 2\epsilon_\tau$	$\frac{(n-4)\epsilon + 24\epsilon_\tau}{n+8}$	$\frac{(n-4)(\epsilon - 2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$
6	$-\epsilon + 2\epsilon_\tau$	$\frac{(n-4)\epsilon + 24\epsilon_\tau}{n+8}$	$\frac{(n-4)(\epsilon - 2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$
7	$-\epsilon + 2\epsilon_\tau$	$\frac{(n-4)\epsilon + 24\epsilon_\tau}{n+8}$	$\frac{(n-4)(\epsilon - 2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$
8	$-\epsilon + 2\epsilon_\tau$	$\frac{(n-4)\epsilon + 24\epsilon_\tau}{n+8}$	$\frac{(n-4)(\epsilon - 2\epsilon_\tau)}{n+8}$	$-\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$	$\frac{(4-n)(\epsilon - 2\epsilon_\tau)}{n+8}$
9	$\frac{\epsilon + 4\epsilon_\tau}{4}$	$\lambda_2^{(9)}$	$\lambda_3^{(9)}$	$\frac{3\epsilon - 4\epsilon_\tau}{4}$	$-\frac{3\epsilon + 4\epsilon_\tau}{4}$
10	$\frac{\epsilon + 4\epsilon_\tau}{4}$	$\lambda_2^{(9)}$	$\lambda_3^{(9)}$	$\frac{3\epsilon - 4\epsilon_\tau}{4}$	$\frac{3\epsilon - 4\epsilon_\tau}{4}$
11	$\frac{\epsilon + 4\epsilon_\tau}{4}$	$\lambda_2^{(9)}$	$\lambda_3^{(9)}$	$-\frac{3\epsilon + 4\epsilon_\tau}{4}$	$-\frac{3\epsilon + 4\epsilon_\tau}{4}$
12	$\frac{\epsilon + 4\epsilon_\tau}{4}$	$\lambda_2^{(9)}$	$\lambda_3^{(9)}$	$-\frac{3\epsilon + 4\epsilon_\tau}{4}$	$\frac{3\epsilon - 4\epsilon_\tau}{4}$
13	$\lambda_1^{(13)}$	$\lambda_2^{(13)}$	$\frac{(n-4)(\epsilon + 4\epsilon_\tau)}{2(10-n)}$	$\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$	$-\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$
14	$\lambda_1^{(13)}$	$\lambda_2^{(13)}$	$\frac{(n-4)(\epsilon + 4\epsilon_\tau)}{2(10-n)}$	$\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$	$\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$
15	$\lambda_1^{(13)}$	$\lambda_2^{(13)}$	$\frac{(n-4)(\epsilon + 4\epsilon_\tau)}{2(10-n)}$	$-\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$	$-\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$
16	$\lambda_1^{(13)}$	$\lambda_2^{(13)}$	$\frac{(n-4)(\epsilon + 4\epsilon_\tau)}{2(10-n)}$	$-\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$	$\frac{(4-n)3\epsilon - (n+8)4\epsilon_\tau}{2(10-n)}$

Fig. 2. Phase diagram for the constrained system ($\Delta^* = 0$, $\bar{x}^* = 0$)

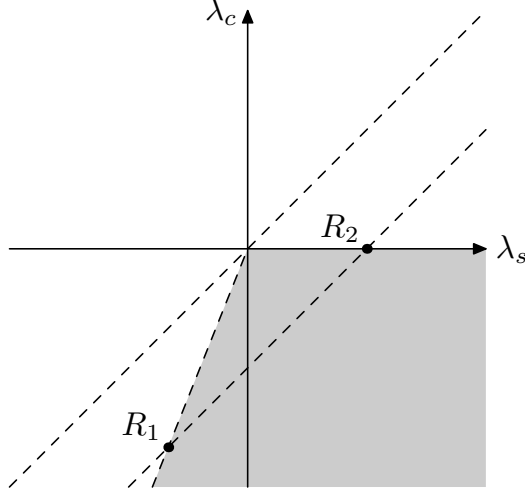


Fig. 3. Phase diagram for the unconstrained system ($\Delta^* \neq 0$, $\bar{x}^* = 0$)

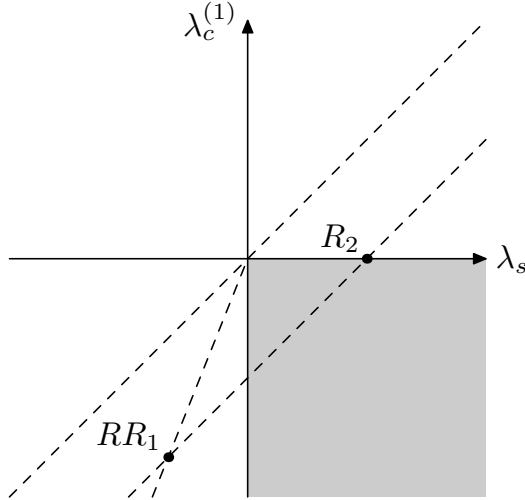


Fig. 4. Phase diagram for the constrained system ($\Delta^* \neq 0$, $\bar{x}^* = 0$)

- (1) $\lambda_s - \lambda_c = (\epsilon + 4\epsilon_\tau)/16K_4(n-1)$ for the unconstrained system ($z = 0$);
- (2) $\lambda_s - \lambda_c^{(1)} = (\epsilon + 4\epsilon_\tau)/16K_4(n-1)$ for the constrained system ($w = 0$).

For the unconstrained system the fixed points lying on these lines for both the classical ($\epsilon_\tau = 0$) and the quantum phase transition are situated in the stability range (Fig. 3). Thus the accessibility range of fixed points is also reduced in comparison with the mean-field result. For the constrained system not all points are in the range of stability (Fig. 4).

The analysis of the renormalization group equations (4), (6) and (5) shows that as well as in the case of systems without hidden degrees of freedom [4,5] the Gaussian fixed point G_2 is stable for $d > 4$ and unstable to disorder for $d < 4$. The random fixed point R_2 is stable for $3n\epsilon > 4(n-4)\epsilon_\tau$, $n > 4$ and $12n\epsilon_\tau > (n-4)\epsilon$. It is easy to see that this point is always stable for $d < 4$ and $4 < n < n_c$ where $n_c = 16$ for the particular case $\epsilon_\tau = \epsilon$. It should be

noted here that in the conventional theory this fixed point is stable for $d < 4$ and $n < 4$ [6,8]. For $d < 4$ the stable unphysical fixed point U_2 is inaccessible from the physical range (according to definition the magnitude Δ should be positive) and for $d > 4$ it is unstable. The Heisenberg fixed point H_2 for the pure quantum system becomes unstable because the dynamic critical exponent z reduces the upper critical dimensionality. There also are new fixed points with $\bar{x}^* \neq 0$. As can be seen from Table 4 they are all unstable except the fixed point 14, which is stable for $n < 4$ and $3(4 - n)\epsilon - 4(n + 8)\epsilon_\tau < 0$. However, this fixed point is also inaccessible from physical range since the value of \bar{x}^* is negative.

Thus, we see that the quantum character of the phase transition in systems with hidden degrees of freedom with constraints leads to conclusion that for pure systems ($\Delta^* = \bar{x}^* = 0$) the phase transition is described by the Gaussian fixed point with “classical” critical exponents of the mean-field theory whereas for the disordered systems ($\Delta^* \neq 0$, $\bar{x}^* \neq 0$) the rare regions destroy the phase transition for $n < 4$ similar to systems without hidden degrees of freedom [4,5]. However, for $4 < n < n_c$ the phase transition is described by the random fixed point. Hidden degrees of freedom result in a decrease of the stability range with comparison to the result of the mean-field theory and existence of the range of the phase space in which the flow trajectories runaway from fixed points not intersecting the stability range of the mean-field.

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